

Near Wall Two Phase Flows : Analytical Solutions and Numerical Predictions	العنوان:
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## Abstract

This thesis presents three different approaches for the theoretical study of two-phase flow and heat transfer, namely, analytical study, numerical predictions, and dimensional analysis. In the first approach, exact solutions of momentum and energy equations for laminar two-phase pipe flow is obtained. Extension to turbulent case is also made. When compared with experimental results the analysis shows a close representation of the velocity and temperature fields of gas-solid flows. Next, a numerical solution of two phase flow and heat transfer using minimum number of assumptions is obtained. The turbulence correlations appearing in the transport equations recently modeled by a two-phase  $k-\epsilon$  model have been modified to take into consideration the wall presence and the particle-turbulent eddies interaction. Particle- eddies interaction have been treated with a new approach based on the relative size of the particle and the eddies, taking into account the presence of the wall. Comparing the results with experimental ones shows a better agreement than the previous predictions. Finally, Dimensional analysis is carried out leading to new correlations for friction factor and convective heat transfer coefficient in terms of other flow parameters. In this analysis a large number of experimental data that covered a wide range of flow conditions is used. A comparison with existing dimensional correlations shows that the newly obtained ones produce a better fit of the existing data.

بسم الله الرحمن الرحيم

## ملخص الرسالة

تحليل نظري وعددي وبعدي لسريانات ثنائية الطور المحتويه

تحتوي هذه الرسالة على ثلاث طرق للتنبؤ بالمتغيرات التي تتعلق بالسريانات ثنائية الطور (Two Phase Flows) عبر الأنابيب مع أو بدون انتقال الحرارة من المائع الى جدار الأنبوب .

في الجزء الأول تم معالجة الموضوع بطريقة رياضية نظرية حيث وضعت المعادلات التي تصف حركة الخليط داخل الأنابيب ثم بسطت وحلت لسريانات الطبقيّة (Laminar). وبعد ذلك طور الحل بطريقة جعلته مناسباً لسريانات الاكثر شيوعاً واستعمالاً في التطبيقات العملية وهي السريانات المضطربة (Turbulent) .

لمعالجة هذه السريانات عندما تكون حركة الخليط مصحوبه بانتقال حراره من أو الى الخليط، وضعت معادلات الطاقة ذات العلاقة وحلت تحت فرضيات مبسطة لحالة السريانات الطبقيّة .

في الجزء الثاني من هذه الرسالة تم تطوير معادلة حركة حبيبات معلقة في سائل مضطرب لتصف حركة هذه الحبيبات عند وجودها في مائع يسري داخل أنابيب أو بجانب حاجز مع الأخذ بعين الاعتبار أثر حجم هذه الحبيبات على طبيعة تفاعلها مع المائع وحركتها بالنسبة له . أدخلت هذه التحسينات في معادلات حفظ كمية الحركة والكتلة للخليط وحلت عددياً للحصول على سرعة المائع والحبيبات والجهودات المختلفة ذات العلاقة، وعندما قورنت النتائج بالقياسات المخبرية لوحظ أن هذه النتائج تصف بدقة سرعة المائع والحبيبات بالإضافة الى الجهودات الأخرى.

وفي الجزء الثالث أستخدمت طريقة التحليل البعدي أو النمف تجريبي للتنبؤ بمعامل الضياعات الاحتكاكية ومعامل انتقال الحرارة جراء سريان خليط من المائع والحبيبات الصلبه داخل انابيب عموديه حيث وضعت المتغيرات التي تصف هذه السريانات ومن ثم أستنبطت المجموعات اللابعديه اللازمه واستخدمت نتائج تجارب سابقه في هذا الحقل لايجاد المعهولات التي تربط هذه المجموعات للحصول على معادلات نصف تجريبية تصف معامل الضياعات الاحتكاكية ومعامل انتقال الحرارة لهذه السريانات.

ومن ثم تمت مقارنة نتائج هذه الطرق الثلاث مع نتائج دراسات تجريبية ونظريه مشابهه في هذا الحقل حيث لوحظت دقة وانسجام هذه النتائج مع النتائج التجريبية وتطورها عن الدراسات النظرية السابقه .

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# Nomenclature

## Roman letters

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B	Drag coefficient, constant.
C	Specific heat.
D	Diameter of the pipe.
d	Diameter of solid particle.
E	Eddy diffusivity of heat.
f	Dimensionless velocity function, friction coefficient.
Gz	Graetz number
h	Convective heat transfer coefficient.
K	Effectiveness of momentum transfer.
k	Thermal conductivity.
L	Length of pipe.
l	Micro length scale.
M	Mass.
m	Mass of solid Particle.
N	Number density of solid particles.
Nu	Nusselt number.
Q	Heat transfer rate.
P	Pressure.
Pr	Prandtl number.
q	Heat flux.
r	Radial distance.
R	Radius of the pipe.
Re	Reynolds number.
U	Axial velocity component.
U0	Inlet Velocit of fluid phase.
u	Axial fluctuating velocity component.
T	Temperature.
t	time.
V	Radial velocity component.
v	Radial fluctuating velocity component.
W	Tangential azimuthal velocity component.
w	Tangential azimuthal fluctuating velocity component.
x	Axial coordinate.
Y	Dimensionless distance from the wall.
y	Radial coordinate.
z	Axial distance.

## Greek letters

$\alpha$	Constant.
$\beta$	Constant.
$\gamma$	Eigenvalue, constant.
$\eta$	Dimensionless radial coordinate.
$\theta$	Tangential coordinate.
$\Theta$	Dimensionless Temperature.
$\lambda$	Constant.
$\mu$	Dynamic viscosity.
$\nu$	Kinematic viscosity.
$\xi$	Dimensionless axial coordinate.
$\rho$	Density.
$\sigma$	Constant.
$\tau$	Dummy variable.
$\phi$	Volume fraction.
$\psi$	Constant.
$\Omega$	Frequency function
$\omega$	Mass loading ratio, frequency.

## Subscript

a	Air.
avg	Average.
eff	Effective.
l	Laminar.
f	Fluid.
g	Gas.
max	Maximum.
o	Single phase.
p	Particle.
r	radial.
s	Suspension.
T	Turbulent.
x	Axial.
y	Radial.
z	Axial.
w	Wall.
$\theta$	Tangential.
1	Fluid phase.
2	Dispersed, <i>solid</i> , phase.

## Superscript

- h Homogeneous.
- n Fourier component.
- p Particular.
- Time average.
- \* Complex conjugate.
- ~ Fourier transform.



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BY

AHMAD FAWZI AHMAD AL-QASSEM

B.Sc. Mechanical Engineering Yarmouk University 1987

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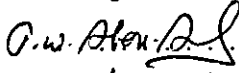
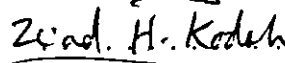


in

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at

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## Abstract

This thesis presents three different approaches for the theoretical study of two-phase flow and heat transfer, namely, analytical study, numerical predictions, and dimensional analysis. In the first approach, exact solutions of momentum and energy equations for laminar two-phase pipe flow is obtained. Extension to turbulent case is also made. When compared with experimental results the analysis shows a close representation of the velocity and temperature fields of gas-solid flows. Next, a numerical solution of two phase flow and heat transfer using minimum number of assumptions is obtained. The turbulence correlations appearing in the transport equations recently modeled by a two-phase  $k-\epsilon$  model have been modified to take into consideration the wall presence and the particle-turbulent eddies interaction. Particle- eddies interaction have been treated with a new approach based on the relative size of the particle and the eddies, taking into account the presence of the wall. Comparing the results with experimental ones shows a better agreement than the previous predictions. Finally, Dimensional analysis is carried out leading to new correlations for friction factor and convective heat transfer coefficient in terms of other flow parameters. In this analysis a large number of experimental data that covered a wide range of flow conditions is used. A comparison with existing dimensional correlations shows that the newly obtained ones produce a better fit of the existing data.

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# Chapter 1

## INTRODUCTION

Multiphase systems consist of mixtures of solid particles, liquid droplets, or gas bubbles in fluids. When the fluid medium is gas, the particulate phase may consist of solid particles, liquid droplets, or both. When the fluid medium is liquid the suspended phase may be solid particles, gas bubbles, or liquid droplets immisible to the fluid phase [30].

The importance of studying the dynamics of multiphase flow systems arises from their numerous applications in various technical, and engineering fields such as pneumatic conveying, fluidized beds, fluid atomization, vapour compression refrigerators, combustion processes, petroleum industries, dust collection systems, metallized propellant rockets, sanitary engineering , nuclear power plants, and air pollution problems.

In studying multiphase systems, three types of interactions must be fully analyzed, and thoroughly understood; these are:

1. The interaction between particles of the suspended phase(s).

2. The interaction between the suspended particles and the fluid medium.
3. The interaction between the various phases and the boundary of the system.

These three types of interactions have made the problem of multiphase flows quite complicated to be treated, both analytically and experimentally. In the present study we will try to predict the variables that govern two phase gas-solid flows, with and without heat transfer. Thus we will restrict our discussion in what follows to this system, paying attention to the phenomenon from a physical point of view.

Historically, the interest in gas-solid suspensions was a result of the need for a heat transfer media, that does not suffer from the serious deficiencies that both gases and liquids have. As a heat transfer media, liquids, generally, have better coolant properties than gases, because of their higher density and thermal conductivity, but unless high pressures and relatively low temperatures are used, change of phase may cause instability and/or burn out. Gases on the other hand, can be carried to high temperatures, however, they have low thermal capacity and low thermal transport properties, which give rise to low heat transfer coefficient.

The influence of suspended particles on the transport characteristics of the suspension, as they are manifested at the wall, has been widely discussed by many authors [30, 9]. An obvious point of agreement is that, the solid phase increases the volumetric thermal capacity of the flow, which in turn, increases the heat transfer rate for the same temperature conditions. Beyond this elementary view, there are numerous explanations. There is generally a consensus that direct heat transfer from the wall to the solid phase is negligible due to the small time and area of contact between the particles and the wall [9]. A consequence of this heat transfer mechanism, is that the temperature of the solids lags the temperature of the gas in the case of heating, and leads in the case of cooling,

assuming that the solids and the gas temperatures have equilibrated prior to the onset of heat transfer process. The gas temperature increases more rapidly than the solids temperature, until some balance of temperature potential and heating rate between the wall and the gas, and between the gas and the solids is achieved. It is reasonable to expect that at some distance from the thermal entrance, the flow will be fully developed thermally, and the two phases will have similar temperature profiles if the thermodynamic and transport properties of the two phases are reasonably constant.

The transfer of heat to the solid particles depend on; the particle geometry, physical properties, and motion relative to the gas. Whereas the net transfer of heat to the gas depends on, the loss of heat to the solids, versus direct gain from the wall through the laminar sublayer, the buffer region, and the turbulent core. Alteration of the laminar or turbulent structure of these regions may profoundly affect the rate of heat transfer to the suspension, and the fluid dynamics of the flow is a basic facet of the convective process. Solids can alter convective heat transfer process in several ways :

1. Penetration of solids through the buffer layer, and into the laminar sublayer would cause a disturbance and thinning of that layer and, consequently, reduction in its resistance to heat transfer.
2. The presence of solids may cause a damping of convective eddies, and a consequent reduction in turbulent transport properties.
3. The slip between the particles and the gas, may enhance the turbulent mixing of the carrier gas.
4. Radial motion of the solids would promote the exchange of energy between the laminar sublayer and the turbulent core.

Since most of the applications encountered in gas-solid flows involved dilute suspensions (i.e. low concentration of solids.), and because of the limited knowledge about the nature of the interactions between the particles the present study will be concerned with dilute gas-solid suspension that is characterized by :-

1. Absence of particle-particle interactions, or it does exist at such a level that doesn't have a remarkable effect on the flow field of the suspension.
2. Homogeneity of flow field, in which the solid particles are uniformly distributed throughout the cross section of the flow field.
3. Undeformable particles upon collisions with the boundary of the system.

The present study will also be restricted to confined two-phase gas-solid flows through circular pipes that are vertically oriented to exclude any asymmetry in the radial distribution of solids, and hence the three dimensionality of the flow governing equations, caused by the transverse gravity force in case of horizontally oriented flows. In addition, the radiative heat transfer will not be considered, which implies moderate levels of temperature potential (less than  $500^{\circ}C$ ).

The present study consists of four parts. In the first part, the momentum and energy equations of laminar two phase gas-solid flow (based on the continuum concept) are stated, discussed, and solved for the velocity and temperature fields respectively. Extension to turbulent case is also made and discussed. Turbulent motion of a swarm of discrete solid particles is considered in part two. The equation of motion of solid particle suspended in turbulent flow is discussed, criticized and its turbulent correlations with the carrier fluid is developed for the considered flow. These results are used to predict the transport variables in turbulent two phase pipe flow via a numerical test case. As a

result of the complexity of the study under consideration, we rely on existing experimental data, and generalize the result concerning the frictional pressure loss, and the heat transfer coefficient using dimensional analysis, in part three. Finally, a general discussion of results along with comparison with experimental data and with similar studies made in this field is presented and conclusions are stated.

## Chapter 2

# ANALYTICAL SOLUTION OF TWO PHASE FLOW AND HEAT TRANSFER PROBLEMS

### 2.1 Introduction

In the general case of single phase flows, velocity and temperature fields are mutually interacting, which means that the temperature distribution depends on the velocity field and conversely, the velocity distribution is affected by the temperature field. In the special case, when bouyancy forces can be neglected, and when the properties of the fluid can be considered independent of temperature, mutual interaction ceases, and the velocity field no longer depends on the temperature field. Although the converse dependence of the temperature field on the velocity field still exists. The above modes of interactions between the temperature and velocity fields will be more involved in the case of two phase

3. The solid particles are distributed uniformly throughout the cross section of the flow system.
4. Interactions can exist between the gas and the particles, and between the two phases and the solid boundaries, but the interaction through the solid particles is absent (Dilute suspensions).
5. The drag on the solid particle is mainly due to the relative velocity between the two phase. Hence, the velocity of each solid particle due to its own thermal state is extremely low, and the particulate phase does not contribute significantly to the static pressure of the system.
6. Due to the inertia of the solid particle one has to identify the velocity of the solid particle and that of the gas separately at any given location in space. At a solid boundary, solid particle may have finite velocity even though the gas phase attained the no-slip condition of zero velocity there [30].
7. When heat transfer is considered, radiation to and among the solid particles have insignificant effect on the rate of heat transfer which implies moderate levels of temperature (less than  $500^{\circ}C$  ).
8. The particle dimensions are small and the thermal conductivity is high enough so that the temperature distribution is uniform throughout the particle.

## 2.3 Laminar Two-Phase Flow

In this section the conservation equations of laminar two-phase flow will be stated, simplified, and solved.

### 2.3.1 Flow Governing Equations

The equations of motion (momentum equations) and the conservation of mass (continuity equations) applied to gas-solid suspensions have been given by many authors [11, 30]. These equations for incompressible Newtonian fluids in cylindrical, polar coordinates read:

radial-component:

$$\begin{aligned} \rho_f \phi_f \left( \frac{\partial V_f}{\partial t} + V_f \frac{\partial V_f}{\partial r} + \frac{W_f}{r} \frac{\partial V_f}{\partial \theta} - \frac{W_f^2}{r} + U_f \frac{\partial V_f}{\partial z} \right) &= -(1 - K \phi_p) \frac{\partial P}{\partial r} \\ + \mu_f \phi_f \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (r V_f) \right) + \frac{1}{r^2} \frac{\partial^2 V_f}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial W_f}{\partial \theta} + \frac{\partial^2 V_f}{\partial z^2} \right] \\ - KF \phi_p (V_f - V_p) + f_r \end{aligned} \quad (2.1)$$

tangential-component

$$\begin{aligned} \rho_f \phi_f \left( \frac{\partial W_f}{\partial t} + V_f \frac{\partial W_f}{\partial r} + \frac{W_f}{r} \frac{\partial W_f}{\partial \theta} + \frac{W_f V_f}{r} + U_f \frac{\partial W_f}{\partial z} \right) &= -(1 - K \phi_p) \frac{1}{r} \frac{\partial P}{\partial \theta} \\ + \mu_f \phi_f \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (r W_f) \right) + \frac{1}{r^2} \frac{\partial^2 W_f}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial V_f}{\partial \theta} + \frac{\partial^2 W_f}{\partial z^2} \right] \\ - KF \phi_p (W_f - W_p) + f_\theta \end{aligned} \quad (2.2)$$

axial-component:

$$\rho_f \phi_f \left( \frac{\partial U_f}{\partial t} + V_f \frac{\partial U_f}{\partial r} + \frac{W_f}{r} \frac{\partial U_f}{\partial \theta} + U_f \frac{\partial U_f}{\partial z} \right) = -(1 - K \phi_p) \frac{\partial P}{\partial z}$$



$$\begin{aligned}
& +\mu_f \phi_f \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial U_f}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 U_f}{\partial \theta^2} \frac{\partial^2 U_f}{\partial z^2} \right] \\
& -KF\phi_p(V_f - V_p) + f_z
\end{aligned} \tag{2.3}$$

For the carrier(lighter) fluid and radial-component

$$\begin{aligned}
\rho_p \phi_p \left( \frac{\partial V_p}{\partial t} + V_p \frac{\partial V_p}{\partial r} + \frac{W_p}{r} \frac{\partial V_p}{\partial \theta} - \frac{W_p^2}{r} + U_p \frac{\partial V_p}{\partial z} \right) &= -\phi_p \frac{\partial P}{\partial r} \\
& +\mu_p \phi_p \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (r V_p) \right) + \frac{1}{r^2} \frac{\partial^2 V_p}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial W_p}{\partial \theta} + \frac{\partial^2 V_p}{\partial z^2} \right] \\
& +KF\phi_p(V_f - V_p) + f_z
\end{aligned} \tag{2.4}$$

tangential-component :

$$\begin{aligned}
\rho_p \phi_p \left( \frac{\partial W_p}{\partial t} + V_p \frac{\partial W_p}{\partial r} + \frac{W_p}{r} \frac{\partial W_p}{\partial \theta} + \frac{W_p V_p}{r} + U_p \frac{\partial W_p}{\partial z} \right) &= -\frac{\phi_p}{r} \frac{\partial P}{\partial \theta} \\
& +\mu_p \phi_p \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (r W_p) \right) + \frac{1}{r^2} \frac{\partial^2 W_p}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial V_p}{\partial \theta} + \frac{\partial^2 W_p}{\partial z^2} \right] \\
& +F\phi_p(W_f - W_p) + f_\theta
\end{aligned} \tag{2.5}$$

axial-component :

$$\begin{aligned}
\rho_p \phi_f \left( \frac{\partial U_p}{\partial t} + V_p \frac{\partial U_p}{\partial r} + \frac{W_p}{r} \frac{\partial U_p}{\partial \theta} + U_p \frac{\partial U_p}{\partial z} \right) &= -\phi_p \frac{\partial P}{\partial r} \\
& +\mu_p \phi_p \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial U_p}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 U_p}{\partial \theta^2} + \frac{\partial^2 U_p}{\partial z^2} \right] \\
& +F\phi_p(V_f - V_p) + f_z
\end{aligned} \tag{2.6}$$

For the particulate(solid) phase.

In addition to the continuity equation of the fluid phase which reads :

$$\frac{\partial \rho_f \phi_f}{\partial t} + \frac{1}{r} \frac{\partial (\rho_f \phi_f r V_f)}{\partial r} + \frac{1}{r} \frac{\partial (\rho_f \phi_f W_f)}{\partial \theta} + \frac{\partial (\rho_f \phi_f U_f)}{\partial z} = 0. \tag{2.7}$$

and for the particulate phase reads:

$$\frac{\partial \rho_p \phi_p}{\partial t} + \frac{1}{r} \frac{\partial (\rho_p \phi_p r V_p)}{\partial r} + \frac{1}{r} \frac{\partial (\rho_p \phi_p W_p)}{\partial \theta} + \frac{\partial (\rho_p \phi_p U_p)}{\partial z} = 0. \tag{2.8}$$

Where the global continuity equation reads:

$$\phi_f + \phi_p = 1 \quad (2.9)$$

Where  $\phi$  denotes the volume fraction of solids, and  $V, W$ , and  $U$  are the component of velocity vector in the radial, tangential, and axial directions respectively. The subscripts (f) and (p) denote the fluid and the solid phase, and  $K$  is the local effectiveness of the momentum transfer from the dispersed phase to the fluid phase and, it is discussed in detail by Soo.[30]. It suffices here to state that  $K$  equals unity in this simplified case.

For one dimensional flow the radial and tangential component of the velocity of both phases are identically zero, and the steady flow condition implies ( $\frac{\partial}{\partial t} = 0$ ). Applying these conditions to the momentum and continuity equations of both phases we obtain :

$$\phi_f \frac{\partial P}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \lambda_f U_f}{\partial r} \right) - \phi_p F(U_f - U_p) \quad (2.10)$$

for the fluid phase, and

$$\phi_p \frac{\partial P}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \lambda_p U_p}{\partial r} \right) + \phi_p F(U_f - U_p) \quad (2.11)$$

for the solid phase.

Where the continuity equation for the fluid phase becomes

$$\frac{\partial}{\partial z} (\rho_f \phi_f U_f) = 0 \quad (2.12)$$

where

$$\lambda_f = \mu_f \phi_f$$

and

$$\lambda_p = \mu_p \phi_p$$

In addition to the global continuity equation (equation 2.9).

Equations 2.9, 2.10, and 2.11 describe the laminar one dimensional steady flow which we have to solve along with the following boundary conditions

$$\begin{aligned} U_f(r = R) &= 0 \\ \frac{\partial U_f}{\partial r}(r = 0) &= 0 \\ U_p(r = R) &= U_p^* \\ \frac{\partial U_p}{\partial r}(r = 0) &= 0 \end{aligned} \tag{2.13}$$

where  $U_p^*$  is the velocity that the solid particles slides on the wall by.

### 2.3.2 Method of Solution

To solve the system of equations 2.9, 2.10, and 2.11 we proceed as follows:

Eliminating the drag term  $\phi_p F(U_f - U_p)$  between equations 2.10 and 2.11 we have :

$$(1 - \phi_p) \frac{\partial P}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \lambda_f U_f}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \lambda_p U_p}{\partial r} \right) - \phi_p \frac{\partial P}{\partial z} \tag{2.14}$$

rearranging equation 2.14 we get

$$\frac{\partial P}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left[ r \frac{\partial}{\partial r} (\lambda_f U_f + \lambda_p U_p) \right] \quad (2.15)$$

Now integrating equation 2.15 with respect to  $r$  for constant pressure gradient ( $\frac{\partial p}{\partial z} = \text{constant}$ .) we obtain :

$$\frac{\partial}{\partial r} (\lambda_f U_f + \lambda_p U_p) = \frac{\partial p}{\partial z} \frac{r}{2} + \frac{C_1}{r} \quad (2.16)$$

Where  $C_1$  is a *constant* of integration.

Integrating equation 2.16 once more we have

$$\lambda_f U_f + \lambda_p U_p = \frac{\partial p}{\partial z} \frac{r^2}{4} + C_1 \ln r + C_2 \quad (2.17)$$

Where  $C_2$  is another *constant* of itegration that will be determined from the boundary conditions.

Applying the second and the last boundary conditions in equation 2.13 we obtain

$$C_1 = 0. \quad (2.18)$$

and from the first and third boundary conditions we have

$$C_2 = \lambda_p U_p^* - \frac{R^2}{4} \frac{\partial p}{\partial z} \quad (2.19)$$

Finally solving for the fluid phase velocity ( $U_f$ ) we have

$$U_f = \frac{\partial P}{\partial z} \frac{1}{4\lambda_f} r^2 + \frac{C_2}{\lambda_f} - \frac{\lambda_p}{\lambda_f} U_p \quad (2.20)$$

Substituting for  $U_f$  into equation 2.10 from equation 2.20 we readily obtain

$$\frac{\partial^2 U_p}{\partial r^2} + \frac{1}{r} \frac{\partial U_p}{\partial r} - \frac{F \phi_p}{\lambda_p} \left( 1 + \frac{\lambda_p}{\lambda_f} \right) U_p = \frac{\partial P \phi_p}{\partial z \lambda_p} - \frac{\phi_p F C_2}{\lambda_f \lambda_p} - \frac{F \phi_p \frac{\partial P}{\partial z}}{4 \lambda_f \lambda_p} r^2 \quad (2.21)$$

Which may be rewritten as

$$\frac{\partial^2 U_p}{\partial r^2} + \frac{1}{r} \frac{\partial U_p}{\partial r} - \gamma^2 U_p = G - \psi r^2 \quad (2.22)$$

Where

$$\gamma^2 = \frac{F \phi_p}{\lambda_p} \left( 1 + \frac{\lambda_p}{\lambda_f} \right) \quad (2.23)$$

$$\psi = \frac{F \phi_p \frac{\partial P}{\partial z}}{4 \lambda_f \lambda_p} \quad (2.24)$$

$$G = \frac{\partial P \phi_p}{\partial z \lambda_p} - \frac{\phi_p F C_2}{\lambda_f \lambda_p} \quad (2.25)$$

The homogeneous part of equation 2.22 has the following solution

$$U_p^h = C_3 I_0(\gamma r) + C_4 K_0(\gamma r) \quad (2.26)$$

Where  $I_0$ , and  $K_0$  are the Modified Bessel Functions of the first and second kind of zero order respectively, and  $C_3$  and  $C_4$  are *constants* of integration.

Applying the fourth boundary condition of equation 2.12 to equation 2.26 we obtain

$$C_4 = 0. \quad (2.27)$$

The particular solution of equation 2.22 reads

$$U_p^p = C_5 r^2 + C_6 \quad (2.28)$$

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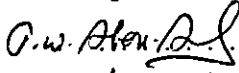
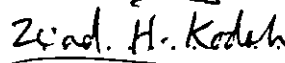


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